SN – 355

V Semester B.A./B.Sc. Examination, November/December 2017
(Fresh + Repeaters) (CBCS) (2016-17 and Onwards)
MATHEMATICS – VI

Time : 3 Hours   Max. Marks : 70

***Instruction** : Answer **all** questions.*

PART – A

Answer **any five** questions. **(5×2=10)**

1. a) Write the Euler's equation when f is independent of x.

b) Find the differential equation in which functional $\int_{x_1}^{x_2} (y^2 + x^2 y^1)\, ds$ assumes extreme values.

c) Define Geodesic on a surface.

d) Show that $\int_c (x+y)dx + (x-y)dy = 0$ where 'c' is simple closed path.

e) Evaluate $\int_0^a \int_0^b (x^2 + y^2)\, dx\, dy$.

f) Evaluate $\int_0^1 \int_0^2 \int_0^3 (x+y+z)\, dx\, dy\, dz$.

g) State Stoke's theorem.

h) Using Green's theorem show that the area bounded by simple closed curve C is given by $\int_C xdy - ydx$.

P.T.O.

PART – B

(2×10=20)

Answer **two full** questions.

2. a) Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.

b) Show that the equation of the curve joining the points (1, 0) and (2, 1) for $I = \int_1^2 \frac{1}{x}\sqrt{1+(y')^2}\, dx$ is a circle.

OR

3. a) Show that the general solution of the Euler's equation for the integral $I = \int_{x_1}^{x_2} \left(\frac{y'}{y}\right)^2 dx$ is $y = ae^{bx}$.

b) Find the Geodesic on a surface of right circular cylinder.

4. a) If cable hangs freely under gravity from two fixed points, show that the shape of the curve is catenary.

b) Find the extremal of the functional $I = \int_0^{\pi} \left((y')^2 - y^2\right) dx$ under the conditions $y = 0, x = 0, x = \pi, y = 1$ subject to the condition $\int_0^{\pi} y\, dx = 1$.

OR

5. a) Find the extremal of the integral $I = \int_0^1 (y')^2 dx$ subject to the constraint $\int_0^1 y dx = 1$ and having $y(0) = 0, y(1) = 1$.

b) Find the extremal of the functional $\int_{x_1}^{x_2} \left(y^2 + (y')^2 + 2ye^x\right) dx$.

PART – C

Answer **two full** questions. **(2×10=20)**

6. a) Evaluate $\int_c (x^2 + 2y^2x)dx + (x^2y^2 - 1)dy$ around the boundary of the region defined by $y^2 = 4x$ and $x = 1$.

   b) Evaluate $\iint_R (x^2 + y^2)dydx$ over the region in the positive quadrant for which $x + y \leq 1$.

OR

7. a) Evaluate $\int_0^a \int_0^{2\sqrt{ax}} x^2 dydx$ by changing the order of integration.

   b) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration.

8. a) Evaluate $\int_{-a}^{a} \int_{-b}^{b} \int_{-c}^{c} (x^2 + y^2 + z^2) dz\,dy\,dx$.

   b) By changing into polar co-ordinates, evaluate $\iint_R \sqrt{x^2 + y^2}\, dxdy$, where R is a circle $x^2 + y^2 = a^2$.

OR

9. a) Find the volume bounded by the surface $z = a^2 - x^2$ and the planes $x = 0$, $y = 0$, $z = 0$, $y = b$.

   b) Evaluate $\iiint_R xyz\,dx\,dy\,dz$ by changing it to the cylindrical polar coordinates where R is region bounded by the planes $x = 0$, $y = 0$, $z = 0$, $z = 1$ and the cylinder $x^2 + y^2 = 1$.

PART – D

(2×10=20)

Answer **two full** questions.

10. a) Evaluate using Green's theorem in the plane for $\int_c \left(3x^2 - 8y^2\right)dx + (4y - 6xy)\,dy$, where 'c' is boundary of the region enclosed by $x = 0$, $y = 0$ and $x+y = 1$.

b) Using Gauss-divergence theorem, show that :

i) $\iint_s \vec{r}.\hat{n}\,ds = 3v$ ii) $\iint_s \nabla r^2 \hat{n}\,ds = 6v$.

OR

11. a) State and prove Green's theorem.

b) Using Gauss-divergence theorem. Evaluate $\iint_s \vec{F}.\hat{n}\,ds$ where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and s is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

12. a) Verify Stoke's theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where s is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

b) Using Gauss divergence theorem evaluate $\iint_s \left(x\hat{i} + y\hat{j} + z^2\hat{k}\right).\hat{n}\,ds$ where s is closed surface bounded by cone $x^2 + y^2 = z^2$ and plane $z = 1$.

OR

13. a) Evaluate by Stoke's theorem $\int_c \sin z\,dx - \cos x\,dy + \sin y\,dz$, c is the boundary of the rectangle $0 \le x \le \pi, 0 \le y \le 1, z = 3$.

b) Verify Green's theorem for $\int_c \left(xy + y^2\right)dx + x^2dy$ where c is the closed curve bounded by $y = x$ and $y = x^2$.